

Keeping in mind the above, the endogeneity of x_{2i} can be tested provided we assume that the instrument z_{2i} is valid. Hausman (1978) proposes to compare the OLS and IV estimators for β . Assuming $E\{\varepsilon_i z_{2i}\} = 0$, the IV estimator is consistent. If, in addition, $E\{\varepsilon_i x_{2i}\} = 0$, the OLS estimator is also consistent and should differ from the IV one by sampling error only. A computationally attractive version of the Hausman test for endogeneity (often referred to as the Durbin–Wu–Hausman test) can be based upon a simple auxiliary regression. First, estimate a regression explaining x_{2i} from x_{1i} and z_{2i} , and save the residuals, say \hat{v}_i . This is the reduced form equation. Next, add the residuals to the model of interest and estimate

$$y_i = x'_{1i}\beta_1 + x_{2i}\beta_2 + \hat{v}_i\gamma + e_i$$

by OLS. This reproduces⁷ the IV estimator for β_1 and β_2 , but also produces an estimate for γ . If $\gamma = 0$, x_{2i} is exogenous. Consequently, we can easily test the endogeneity of x_{2i} by performing a standard t -test on $\gamma = 0$ in the above regression. Note that the endogeneity test requires the assumption that the instrument is valid and therefore does not help to determine which identifying moment condition, $E\{\varepsilon_i x_{2i}\} = 0$ or $E\{\varepsilon_i z_{2i}\} = 0$, is appropriate.